



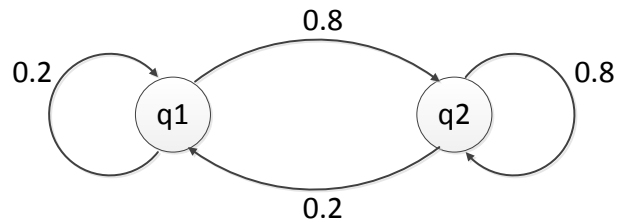
Simulation Systems Practice Exam May 2012

Solve As Much As You Can – Maximum Grade: 40 Points

- [3 points] Write the algorithm that implements the following random number generator:
$$\text{Ran} = A2_r(C1) \wedge B9$$
- [15 points] Mark the following questions as either True or False and give reasons:
 - Good random number generators may not use LCG parts in their formulas.
 - Comparison function in transformation method must have an indefinite integral that is a strict upper bound to desired distribution.
 - It is possible to use rejection method to generate samples from discrete distributions.
 - Simulated annealing optimization is a direct application of Metropolis sampling.
 - Gibbs sampling is similar in steps to importance sampling but with different weights.
 - For a Markov model, if the transition matrix does not have at least one nonzero eigenvalues, then the model does not have a stationary distribution.
 - Metropolis Markov models always have unique stationary distributions.
- [3 points] Is it always possible to use the Rejection method to obtain samples from any distribution? Explain your answer.
- [5 points] In random sampling using the rejection method, the first random deviate was obtained from a uniform distribution $\sim[0, 100]$ and came out to be 67. This corresponds to a value of $x=3.1$ with value of desired probability density function $p(x)=0.2$ and with value of comparison function $f(x)=0.5$. If the second random deviate was found to be 0.3, determine whether x will be accepted or rejected and why.
- [5 points] In the rejection method, if the area under the comparison function $f(x)$ is 2.1 such that it upper bounds the desired distribution $p(x)$, determine how many uniform samples are required to generate 1000 samples from the desired distribution $p(x)$.
- [4 points] For a Markov model with eigenvalues of 1, 0.9, 0.5 and 0.4, determine whether or not the model has stationary states.

7. [5 points] For the shown Markov model, determine if it is ergodic or not and write its transition matrix P. If the initial distribution is $p=[1 \ 0]$, compute 3 time steps of this model. If the output matrix is given as below, determine the outputs of the system in the simulated 3 steps.

$$B = \begin{bmatrix} 0.8 & 0.1 & 0.1 \\ 0.2 & 0.4 & 0.4 \end{bmatrix}$$



Best of Luck