

MULTIDIMENSIONAL SIGNAL PROCESSING (BIO442) FINAL EXAM

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**SOLVE AS MUCH AS YOU CAN – MAXIMUM GRADE: 75 POINTS**

**QUESTION GROUP I: CHOOSE THE BEST ANSWER FOR THE FOLLOWING PROBLEMS [3 POINTS EACH]:**

1. The orthogonality of the Fourier transformation means that ...
  - (a) It preserves distance
  - (b) It preserves angle
  - (c) It preserves both distance and angle (\*)
  
2. The ... theorem can be used to make CT reconstruction simpler.
  - (a) projection-slice (\*)
  - (b) parseval's
  - (c) sampling
  
3. A complex signal with a center frequency of 5 MHz and bandwidth of 10 kHz can be sufficiently sampled with the minimum sampling rate of ...
  - (a) 10.01 M Samples/s
  - (b) 10 M samples/s
  - (c) 10 k Samples/s (\*)
  
4. Time delay between two signals can be computed using the following Fourier transformation property ...
  - (a) convolution
  - (b) time-shift (\*)
  - (c) uncertainly principle
  
5. In ultrasound image reconstruction, signal processing is involved in ...
  - (a) changing of sampling rate in two dimensions (\*)
  - (b) demodulation of the signal
  - (c) logarithmic compression
  
6. In magnetic resonance imaging, the image reconstruction is simply ...
  - (a) an interpolation
  - (b) a sampling rate change
  - (c) an inverse 2D Fourier transformation (\*)
  
7. Linearity of a system is equivalent to the property of ...
  - (a) superposition (\*)
  - (b) homogeneity
  - (c) additivity
  
8. In Doppler ultrasound, the output can be in the form of a ... of the Doppler shifted signal.
  - (a) spectrogram (\*)

- (b) power spectrum
- (c) Fourier transform

9. DFT is a Fourier transformation with assumed ...

- (a) periodicity in time domain
- (b) periodicity in the frequency domain
- (c) periodicity in both time and frequency domains (\*)

10. To compute the correct linear convolution of two sequences of length N and M using DFT, one must ...

- (a) zero pad one of them to have both sequences with the same length
- (b) zero pad both sequences to at least M+N-1 (\*)
- (c) avoid circular convolution

11. Input/output relationship in linear systems is generally defined by ...

- (a) transfer function
- (b) superposition integral (\*)
- (c) convolution

12. If the desired frequency domain resolution is 0.1 Hz, then we must ensure that ...

- (a) sampling duration is 10 s (\*)
- (b) sampling rate is 0.1 samples/s
- (c) sampling rate is 10 samples/s

13. The fundamental period of the signal  $x(n) = \exp(j \pi n/1.1) + \exp(j 2 \pi n/3.3)$  is

- (a) 33 (\*)
- (b) 3.3
- (c)  $\infty$

The correct period is 66. This problem will count as “correct” for all students regardless of their answers.

14. Periodogram averaging with N overlapped windows improves power spectrum signal-to-noise ratio by a factor of ...

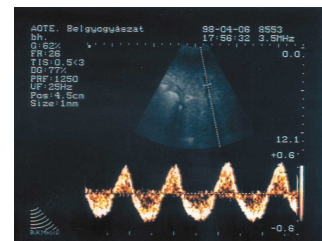
- (a)  $\sqrt{N}$
- (b) greater than  $\sqrt{N}$
- (c) less than  $\sqrt{N}$  (\*)

15. Multiplying a discrete signal by  $(-1)^n$  results in ...

- (a) shift in frequency domain (\*)
- (b) convolution in the frequency domain with a Sinc function
- (c) reduction of the signal energy according to Parseval's theorem

16. The image shown to the right represents:

- (a) Doppler spectrogram (\*)
- (b) Doppler power spectrum
- (c) Doppler frequency domain



**Question Group II:**

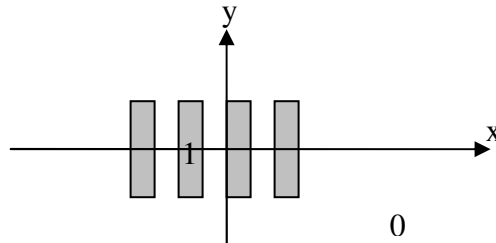
**Label the following Statements as either True or False and explain your answer [1 point each]:**

1. Changing the sampling by a factor of  $\frac{1}{2}$  results in loss of information. (T)
  2. The DFT of a signal can be computed using the continuous Fourier transformation (I)
  3. Linear convolution cannot be computed using the DFT. (F)
  4. Stability of a digital system means no overflow or underflow in computations (I)
  5. Power spectrum estimation is best used for signals with time-varying frequency content. (F)
  6. Spectrograms are two dimensional images of the 2D Fourier transformation (F)
  7. It is possible to completely recover an analog signal from its digital samples if the sampling rate was higher than the bandwidth of the signal. (T)
  8. According to Parseval's theorem, energy is the same in both time and frequency domains. (T)
  9. To recover an analog signal from its discrete samples, a digital low-pass filter must be used. (F)
  10. The theory of finite impulse response digital filters relies on analog filter design methods. (F)
  11. Spectral leakage and Gibbs ringing can be significantly reduced by windowing. (T)
  12. One can compute DTFT on a computer. (F)
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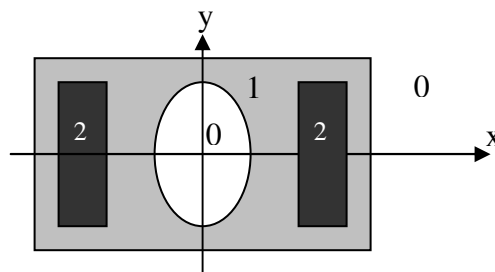
**Question Group III:**

**Compute the Fourier transformation for the following shapes [ 6 points each]:**

- a. Element width: 1mm, element height: 3 mm, inter-element spacing: 1mm.



- b. Outer rectangle: 8 mm  $\times$  4 mm centered at (0,0), inner rectangles: 3 mm  $\times$  1 mm centered at (-3,0) and (3,0), ellipse axes: 3 mm  $\times$  2 mm centered at (0,0).



**Question Group IV: Design Problems**

*(Note: design output should be in the form of a clearly labeled block diagram and detailed specifications for each block)*

**IV.1** [6 points] In a biomedical system, it is desired to calculate an accurate power spectrum for a biomedical signal. Assume that the sampling rate is 10 kHz and the number of samples acquired is 1000

samples. Design an algorithm to calculate the power spectrum of the data given that the desired frequency domain resolution is 100 Hz. Make sure that the SNR of the resultant power spectrum is optimal (i.e., use your best averaging strategy).

Solution: periodogram averaging – overlapping or nonoverlapping should be fine. Just write the steps to perform this.

**IV.2** [6 points] In a PET system, it is desired to calculate an accurate estimate of the delay between two signals. Assume that the sampling rate is 1 GHz and the number of samples acquired is  $10^6$  samples. The desired delay estimation accuracy is 0.1 ns. Design a digital signal processing system to calculate this delay.

Solution: Delay estimation can be done with Fourier shift theorem between the two signals. The difference in Fourier phase represents a linear curve in that delay.

**IV.3** [6 points] In an automated ECG arrhythmia detection, it is desired to take a record of finite number of samples (say 3 s) and process it to generate descriptive features that can be used for classification. The ECG records available to train the system have signals with a sampling rate of 250 Hz and others with a sampling rate of 360 Hz. It is desired to use all records together to construct the ECG classifier. Design a signal processing system that enables all samples to be converted to the same sampling rate without any loss of information in the existing samples.

Solution: convert 250Hz to 360Hz (other way around is incorrect) using upsampling then downsampling.

**Best of Luck!**